CHAPTER - 21
SURFACE AREAS & VOLUMES

EXERCISE 21 (A)

Question 1.
Find the volume and the total surface area of a cuboid, whose :
(i) length = 15 cm, breadth = 10 cm and height = 8 cm.
(ii) l = 3.5 m, b = 2.6 m and h = 90 cm,
Solution:
(i) Length =15 cm, Breadth = 10 cm, Height = 8 cm.
Volume of a cuboid = Length x Breadth x Height = 15 x 10 x 8 =1200 cm^3.
Total surface area of a cuboid 2 (l x b + b x h + h x l) = 2 (15 x 10 + 10 x 8 + 8 x 15) = 2(150 + 80 +120) = 2 x 350 = 700 cm^2
(ii) Length = 3.5 m Breadth = 2.6 m, Height = 90 cm = $\frac{90}{100}$ m = 0.9 m.
Volume of a cuboid = l x b x h = 3.5 x 2.6 x 0.9 = 8.19 m^3
Total surface area of a cuboid = 2(l x b + b x h + h x l)
= 2 (3.5 x 2.6 + 2.6 x 0.9 + 0.9 x 3.5) = 2 (910 + 2.34 + 3.15) = 2(14.59)= 29.18 m^2

Question 2.
(i) The volume of a cuboid is 3456 cm^3. If its length = 24 cm and breadth = 18 cm ; find its height.
(ii) The volume of a cuboid is 7.68 m^3. If its length = 3.2 m and height = 1.0 m; find its breadth.
(iii) The breadth and height of a rectangular solid are 1.20 m and 80 cm respectively. If the volume of the cuboid is 1.92 m^3; find its length.
Solution:
(i) Volume of the given cuboid = 3456 cm^3.
Length of the given cuboid = 24 cm.
Breadth of the given cuboid = 18 cm
We know,
Length x Breadth x Height = Volume of a cuboid
⇒ 24 x 18 x Height = 3456
⇒ Height = $\frac{3456}{24\times18}$
⇒ Height = $\frac{3456}{432}$
⇒ Height = 8 cm
(ii) Volume of a cuboid = 7.68 m^3
Length of a cuboid = 3.2 m
Height of a cuboid = 10m
We know
Length x Breadth x Height = Volume of a cuboid
3.2 x Breadth x 1.0 = 7.68
⇒ Breadth = $\frac{7.68}{3.2\times1.0}$

⇒ Breadth = $\frac{7.68}{3.2}$
⇒ Breadth = 2.4 m
(iii) Volume of a rectangular solid = 1.92 m^3
Breadth of a rectangular solid = 1.20 m
Height of a rectangular solid = 80 cm = 0.8 m
We know
Length x Breadth x Height = Volume of a rectangular solid (cubical)
Length x 1.20 x 0.8 = 1.92
Length x 0.96 = 1.92
⇒ Length = $\frac{1.92}{0.96}$
⇒ Length = $\frac{192}{96}$
⇒ Length = 2 m

Question 3.
The length, breadth and height of a cuboid are in the ratio 5 : 3 : 2. If its volume is 240 cm^3; find its dimensions. (Dimensions means : its length, breadth and height). Also find the total surface area of the cuboid.
Solution:
Let length of the given cuboid = 5x
Breadth of the given cuboid = 3x
Height of the given cuboid = 2x
Volume of the given cuboid = Length x Breadth x Height
= 5x x 3x x 2x = $30x^3$
But we are given volume = 240 cm^3
$30x^3$ = 240 cm^3
⇒ x^3 = $\frac{240}{30}$
⇒ x^3 = 8
⇒ x = $8^{\frac{1}{3}}$
⇒ x = $(2 \times 2 \times 2)^{\frac{1}{3}}$
⇒ x = 2 cm
Length of the given cube = 5x = 5 x 2 = 10 cm
Breadth of the given cube = 3x = 3 x 2 = 6 cm
Height of the given cube = 2x = 2 x 2 = 4cm
Total surface area of the given cuboid = 2(l x b + b x h + h x l)
= 2(10 x 6 + 6 x 4 + 4 x 10) = 2(60 + 24 + 40) = 2 x 124 = 248 cm^2

Question 4.
The length, breadth and height of a cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm2; find its dimensions. Also, find the volume of the cuboid.
Solution:
Let length of the cuboid = 6x
Breadth of the cuboid = 5x
Height of the cuboid = 3x
Total surface area of the given cuboid = 2 (l x b + b x h + h x l)

= 2(6x x 5x + 5x x 3x + 3x x 6x) = 2(30×2 + 15×2 + 18×2)
= 2 x 63×2 = $126x^2$
But we are given total surface area of the given cuboid = 504 cm^2
$126x^2$ = 504 cm^2
=> $x^2 = \frac{504}{126}$
=> x^2 = 4
=> x = √4
=> x = 2 cm.
Length of the cuboid = 6x = 6 x 2 = 12 cm
Breadth of the cuboid = 5x = 5 x 2 = 10cm
Height of the cuboid = 3x = 3 x 2 = 6 cm
Volume of the cuboid = l x b x h = 12 x 10 x 6 = 720 cm^3

Question 5.
Find the volume and total surface area of a cube whose each edge is :
(i) 8 cm
(ii) 2 m 40 cm.
Solution:
(i) Edge of the given cube = 8 cm
Volume of the given cube = $(Edge)^3$ = $(8)^3$ = 8 x 8 x 8 = 512 cm^3
Total surface area of a cube = $6(Edge)^2$ = 6 x $(8)^2$ = 384 cm^2
(ii) Edge of the given cube = 2 m 40 cm = 2.40 m
Volume of a cube = $(Edge)^3$
Volume of the given cube = $(2.40)^3$ = 2.40 x 2.40 x 2.40 = 13.824 m^2
Total surface area of the given cube = 6 x 2.4 x 2.4 = 34.56 m^2

Question 6.
Find the length of each edge of a cube, if its volume is :
(i) 216 cm^3
(ii) 1.728 m^3

Solution:

(i) $(\text{Edge})^3 = \text{Volume of a cube}$

$(\text{Edge})^3 = 216 \text{ cm}^3$

$\Rightarrow \text{Edge} = (216)^{1/3}$

$\Rightarrow \text{Edge} = (3\times3\times3\times2\times2\times2)^{1/3}$

$\Rightarrow \text{Edge} = 3\times2$

$\Rightarrow \text{Edge} = 6 \text{ cm. } \textbf{Ans.}$

(ii) $(\text{Edge})^3 = \text{Volume of a cube}$

$\therefore (\text{Edge})^3 = 1{\cdot}728 \text{ m}^3$

$$\Rightarrow (\text{Edge})^3 = \frac{1{\cdot}728}{1{\cdot}000} = \frac{1728}{1000}$$

$$\Rightarrow \text{Edge} = \left(\frac{1728}{1000}\right)^{1/3}$$

$$\Rightarrow \text{Edge} = \left(\frac{2\times2\times2\times2\times2\times2\times3\times3\times3}{10\times10\times10}\right)^{1/3}$$

$$\Rightarrow \text{Edge} = \frac{2\times2\times3}{10}$$

$$\Rightarrow \text{Edge} = \frac{12}{10}. \text{ m}$$

$\Rightarrow \text{Edge} = 1{\cdot}2 \text{ m } \textbf{Ans.}$

Question 7.

The total surface area of a cube is 216 cm2. Find its volume.

Solution:

$6(\text{Edge})^2$ = Total surface area of a cube

$6(\text{Edge})^2 = 216 \text{ cm}^2$

=> $(\text{Edge})^2 = \frac{216}{6}$

=> $(\text{Edge})^2 = 36$

=> Edge = √36

=> Edge = 6 cm

Volume of the given cube = $(\text{Edge})^3 = (6)^3$ = 6 x 6 x 6 = 216 cm^3

Question 8.

A solid cuboid of metal has dimensions 24 cm, 18 cm and 4 cm. Find its volume.

Solution:

Length of the cuboid = 24 cm

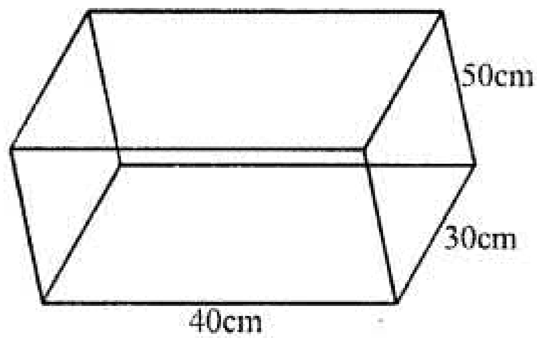
Breadth of the cuboid = 18 cm

Height of the cuboid = 4 cm
Volume of the cuboid = l x b x h = 24 x 18 x 4 = 1728 cm^3

Question 9.
A wall 9 m long, 6 m high and 20 cm thick, is to be constructed using bricks of dimensions 30 cm, 15 cm and 10 cm. How many bricks will be required.
Solution:
Length of the wall = 9 m = 9 x 100 cm = 900 cm
Height of the wall = 6 m = 6 x 100 cm = 600 cm
Breadth of the wall = 20 cm
Volume of the wall = 900 x 600 x 20 cm3 = 10800000 cm^3
Volume of one Brick = 30 x 15 x 10 cm^3 = 4500 cm^3
Number of bricks required to construct the wall = $\frac{Volume \quad of \quad wall}{Volume \quad of \quad one \quad brick}$
= $\frac{10800000}{4500}$
= 2400

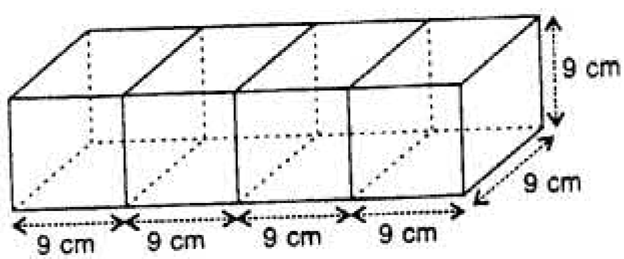
Question 10.
A solid cube of edge 14 cm is melted down and recasted into smaller and equal cubes each of edge 2 cm; find the number of smaller cubes obtained.
Solution:
Edge of the big solid cube = 14 cm
Volume of the big solid cube = 14 x 14 x 14 cm3 = 2744 cm^3
Edge of the small cube = 2 cm
Volume of one small cube = 2 x 2 x 2 cm^3 = 8 cm^3
Number of smaller cubes obtained = $\frac{Volume \quad of \quad big \quad cube}{Volume \quad of \quad one \quad small \quad cube}$
= $\frac{2744}{8}$ = 343

Question 11.
A closed box is cuboid in shape with length = 40 cm, breadth = 30 cm and height = 50 cm. It is made of thin metal sheet. Find the cost of metal sheet required to make 20 such boxes, if 1 m^2 of metal sheet costs Rs. 45.
Solution:
Length of closed box (l) = 40 cm
Breadth (b) = 30 cm
and height (h) = 50 cm



Total surface area = 2 (lb + bh + hl)
= 2 (40 x 30 + 30 x 50 + 50 x 40) cm^2
= 2 (1200 + 1500 + 2000) cm^2
= 2 x 4700 = 9400 cm^2
Surface area of sheet used for 20 such boxes = 9400 x 20 = 188000 cm^2
Cost of 1 m^2 sheet = Rs. 45
Total cost = $\frac{18000 \times 45}{100 \times 100 \times 100}$
= Rs.846

Question 12.
Four cubes, each of edge 9 cm, are joined as shown below :



Write the dimensions of the resulting cuboid obtained. Also, find the total surface area and the volume of the resulting cuboid.
Solution:
Edge of each cube = 9 cm
(i) Length of the cuboid fonned by 4 cubes (l) = 9 x 4 = 36 cm
Breadth (b) = 9 cm and height (h) = 9 cm
(ii) Total surface area of the cuboid = 2(lb + bh + hl)
= 2 (36 x 9 + 9 x 9 + 9 x 36) cm^2
= 2 (324 + 81 + 324) cm^2
= 2 x 729 cm^2
= 1458 cm^2
(iii) Volume = l x b x h = 36 x 9 x 9 cm^2 = 2916 cm^3

EXERCISE 21 (B)

Question 1.
How many persons can be accommodated in a big-hall of dimensions 40 m, 25 m and 15 m ; assuming that each person requires 5 m^3 of air?
Solution:

[No. of persons

$$= \frac{\text{Vol. of the hall}}{\text{Vol. of air required for each person}}]$$

Sol. Length of the hall = 40 m

Breadth ,, ,, ,, = 25 m

Height ,, ,, ,, = 15 m

Volume of the hall = $L \times B \times H$

$= 40 \times 25 \times 15$

$= 15000 \ m^3$

Volume of the air required for each person $= 5m^3$

No. of persons who can be accommodated

$$= \frac{\text{Volume of the hall}}{\text{Volume of air required for each person}}$$

$$= \frac{15000m^3}{5m^3}$$

= 3000 **Ans.**

Question 2.
The dimension of a class-room are; length = 15 m, breadth = 12 m and height = 7.5 m. Find, how many children can be accommodated in this class-room ; assuming 3.6 m^3 of air is needed for each child.
Solution:
Length of the room = 15 m
Breadth of the room = 12 m
Height of the room = 7.5 m
Volume of the room = L x B x H = 15 x 12 x 7.5 m3 = 1350 m^3
Volume of air required for each child = 3.6 m^3

No. of children who can be accommodated in the class room.

$$= \frac{\text{Volume of class room}}{\text{Volume of air needed for each child}}$$

$$= \frac{1350 m^3}{3{\cdot}6 m^3}$$

$$= 375 \textbf{ Ans.}$$

Question 3.
The length, breadth and height of a room are 6 m, 5.4 m and 4 m respectively. Find the area of :
(i) its four-walls
(ii) its roof.
Solution:
Length of the room = 6 m
Breadth of the room = 5.4 m
Height of the room = 4 m
(i) Area of four walls = 2(L+B) x H
= 2(6 + 5.4) x 4 = 2 x 11.4 x 4 = 91.2 m^2
(ii) Area of the roof = L x B = 6 x 5.4 = 32.4 m^2

Question 4.
A room 5 m long, 4.5 m wide and 3.6 m high has one door 1.5 m by 2.4 m and two windows, each 1 m by 0.75 m. Find :
(i) the area of its walls, excluding door and windows ;
(ii) the cost of distempering its walls at the rate of Rs.4.50 per m^2.
(iii) the cost of painting its roof at the rate of Rs.9 per m^2.
Solution:
Length of the room = 5 m
Breadth of the room = 4.5 m

Height of the room = 3.6 m

$$\begin{aligned}
\text{Area of the roof} &= L \times B \\
&= 5 \times 4{\cdot}5 \text{ m}^2 \\
&= 22{\cdot}5 \text{ m}^2 \\
\text{Area of four walls} &= 2[L+B] \times H \\
&= 2[5+4{\cdot}5] \times 3{\cdot}6 \\
&= 2(9{\cdot}5) \times 3{\cdot}6 \\
&= 19 \times 3{\cdot}6 \\
&= 68{\cdot}4 \text{ m}^2 \\
\text{Area of one door} &= 1{\cdot}5 \times 2{\cdot}4 \text{ m}^2 \\
&= 3{\cdot}60 \text{ m}^2 \\
&= 3{\cdot}6 \text{ m}^2 \\
\text{Area of one window} &= 1 \times 0{\cdot}75 \text{ m}^2 \\
&= 0{\cdot}75 \text{ m}^2 \\
\text{Area of 2 windows} &= 0{\cdot}75 \times 2 \text{ m}^2 \\
&= 1{\cdot}5 \text{ m}^2
\end{aligned}$$

(*i*) Area of four walls excluding door and windows

$$\begin{aligned}
&= 68{\cdot}4 - (3{\cdot}6 + 1{\cdot}5) \\
&= 68{\cdot}4 - 5{\cdot}1 \\
&= 63{\cdot}3 \text{ m}^2 \textbf{ Ans.}
\end{aligned}$$

(*ii*) Cost of distempering four walls @ Rs.4·50 per m^2

$$\begin{aligned}
&= 63{\cdot}3 \times 4{\cdot}50 \\
&= \text{Rs.}284{\cdot}85 \textbf{ Ans.}
\end{aligned}$$

(*iii*) Cost of painting the roof @ Rs.9 per m^2

$$\begin{aligned}
&= 22{\cdot}5 \times 9 \\
&= \text{Rs.}202{\cdot}50 \textbf{ Ans.}
\end{aligned}$$

Question 5.
The dining-hall of a hotel is 75 m long ; 60 m broad and 16 m high. It has five – doors 4 m by 3 m each and four windows 3 m by 1.6 m each. Find the cost of :
(i) papering its walls at the rate of Rs.12 per m^2;
(ii) carpetting its floor at the rate of Rs.25 per m^2.

Solution:
Length of the dining hall of a hotel = 75 m
Breadth of the dining hall of a hotel = 60 m
Height of the dining hall of a hotel = 16 m

(i) Area of four walls of the dining hall = 2[L+B) x H = 2(75 + 60) x 16

$$= 2(135) \times 16$$
$$= 270 \times 16$$
$$= 4320 \text{ m}^2$$

Area of one door $= 4 \times 3 \text{ m}^2$
$$= 12 \text{ m}^2$$

Area of 5 doors $= 12 \times 5 = 60 \text{ m}^2$

Area of one window $= 3 \times 1{\cdot}6 = 4{\cdot}8 \text{ m}^2$

Area of 4 windows $= 4{\cdot}8 \times 4 = 19{\cdot}2 \text{ m}^2$

Area of the walls to be papered

$$= 4320 - (60 + 19{\cdot}2)$$
$$= 4320 - 79{\cdot}2$$
$$= 4240{\cdot}8 \text{ m}^2$$

Cost of papering the walls @ Rs.12 per m^2

$$= 4240{\cdot}8 \times 12$$
$$= \text{Rs.}50889{\cdot}60 \text{ Ans.}$$

(*ii*) Area of floor $= \text{L} \times \text{B}$

$$= 75 \times 60$$
$$= 4500 \text{ m}^2$$

Cost of carpetting the floor @ Rs.25 per m^2

$$= 4500 \times 25$$
$$= \text{Rs.}112500 \text{ Ans.}$$

Question 6.

Find the volume of wood required to make a closed box of external dimensions 80 cm, 75 cm and 60 cm, the thickness of walls of the box being 2 cm throughout.

Solution:

External length of the closed box = 80 cm
External Breadth of the closed box = 75 cm
External Height of the closed box = 60 cm
External volume of the closed box = 80 x 75 x 60 = 360000 cm^3
Internal length of the closed box = 80 – 4 = 76 cm
Internal Breadth of the closed box = 75 – 4 = 71 cm
Internal Height of the closed box = 60 – 4=56 cm
Internal volume of the closed box = 76 x 71 x 56 cm = 302176 cm^3
Volume of wood required to make the closed box = 360000 – 302176 = 57824 cm^3

Question 7.

A closed box measures 66 cm, 36 cm and 21 cm from outside. If its walls are made of metal-sheet, 0.5 cm thick ; find :

(i) the capacity of the box ;
(ii) volume of metal-sheet and
(iii) weight of the box, if 1 cm^3 of metal weights 3.6 gm.

Solution:

External length of the closed box = 66cm.
External breadth of the closed box = 36 cm
External height of the closed box =21 cm
External volume of the closed box= 66 x 36 x 21 = 49896 cm^3
Internal length of the box =(66 – 2 x 0.5) = 66 – 1 = 65 cm
Internal breadth of the box =(36 – 2 x 0.5) = 36 – 1 = 35 cm
Internal height of the box = (21 – 2 x 0.5) = 21 – 1 = 20 cm
Internal Volume of the box = 65 x 35 x 20 = 45500 cm^3
(i) Capacity of the box = 45500 cm^3
(ii) Volume of metal sheet of the box = External volume – Internal volume
= 49896 – 45500 = 4396 cm^3
(iii) 1 cm3 of metal weigh 3.6 grams.
Weight of the box = 4396 x 3.6 gm = 15825.6 gm

Question 8.

The internal length, breadth and height of a closed box are 1 m, 80 cm and 25 cm. respectively. If its sides are made of 2.5 cm thick wood ; find :
(i) the capacity of the box
(ii) the volume of wood used to make the box.

Solution:

Internal length of the closed box = 1m = 100 cm
,, breadth ,, ,, ,, ,, = 80 cm
,, height ,, ,, ,, ,, = 25 cm
,, volume ,, ,, ,, ,, = $100 \times 80 \times 25$
= 200000 cm^3

External length of the box = $(100 + 2 \times 2 \cdot 5)$
= 100 + 5 = 105 cm
External breadth ,, ,, ,, = $(80 + 2 \times 2 \cdot 5)$
= 80 + 5 = 85 cm
External height " " " = $(25 + 2 \times 2 \cdot 5)$
= 25 + 5 = 30 cm
External volume ,, ,, ,, = $105 \times 85 \times 30$ cm^3
= 267750 cm^3

(*i*) The capacity of the box

$= 100 \times 80 \times 25 cm^3$

$= 200000 \ cm^3$

$= \frac{200000}{100 \times 100 \times 100} m^3$

$= 0 \cdot 2 \ m^3$ **Ans.**

(*ii*) The volume of wood used to make the box

= External volume – Internal volume

= 267750 – 200000

= 67750 cm^3

$= \frac{67750}{100 \times 100 \times 100} m^3$

$= 0 \cdot 06775 \ m^3$ **Ans.**

Question 9.
Find the area of metal-sheet required to make an open tank of length = 10 m, breadth = 7.5 m and depth = 3.8 m.
Solution:
Length of the tank = 10 m
Breadth of the tank = 7.5 m
Depth of the tank = 3.8 m
Area of four walls = 2[L+B] x H = 2(10 + 7.5) x 3.8
= 2 x 17.5 x 3.8 = 35 x 3.8 = 133 m^2
Area of the floor = L x B = 10 x 7.5 = 75 m
Area of metal sheet required to make the tank =Area of four walls + Area of floor = 133 m^2 + 75 m^2 = 208 m^2

Question 10.
A tank 30 m long, 24 m wide and 4.5 m deep is to be made. It is open from the top. Find the cost of iron-sheet required, at the rate of ₹ 65 per m^2, to make the tank.
Solution:
Length of the tank = 30 m
Width of the tank = 24 m
Depth of the tank = 4.5 m
Area of four walls of the tank = 2[L+B] x H = 2(30 + 24) x 4.5 = 2 x 54 x 4.5 m^2 = 486 m^2
Area of the floor of the tank = L x B = 30 x 24 = 720 m^2
Area of Iron sheet required to make the tank = Area of four walls + Area of floor = 486 + 720 = 1206 m^2
Cost of iron sheet required @ Rs. 65 per m^2 = 1206 x 65 = Rs. 78390

EXERCISE 21(C)

Question 1.

The edges of three solid cubes are 6 cm, 8 cm and 10 cm. These cubes are melted and recast into a single cube. Find the edge of the resulting cube.

Solution:

Edge of first solid cube = 6 cm
Volume = $(6)^3$ = 216 cm^3
Edge of second cube = 8 cm
Volume = $(8)^3$ = 512 cm^3
Edge of third cube = 10 cm
Volume = $(10)^3$ = 1000 cm^3
Sum of volumes of three cubes = 216 + 512 + 1000= 1728 cm^3
Let a be the edge of so formed cube volume = a^3
$a^3 = 1728 = (12)^3$
a = 12 cm

Question 2.

Three solid cubes of edges 6 cm, 10 cm and x cm are melted to form a single cube of edge 12 cm, find the value of x.

Solution:

Edge of first cube = 6 cm
Volume = $(6)^3$ = 216 cm^3
Edge of second cube = 10 cm
Volume = $(10)^3$ = 1000 cm^3
Edge of third cube = x
Volume =x^3
Edge of resulting cube = 12 cm
Volume = $(12)^3$ = 1728 cm^3
$216 + 1000 + x^3 = 1728$
$x^3 = 1728 – 216 – 1000 = 512 = (8)^3$

x = 8
Edge of third cube = 8 cm

Question 3.
The length of the diagonals of a cube is $8\sqrt{3}$ cm.
Find its:
(i) edge
(ii) total surface area
(iii) Volume
Solution:
(i) Length of diagonal of a cube = $8\sqrt{3}$ cm
Length of edge = $\frac{8\sqrt{3}}{\sqrt{3}}$ = 8 cm
(ii) Total surface area = $6a^2 = 6 \times 8^2 = 6 \times 64 \text{ cm}^2 = 384 \text{ cm}^2$
(iii) Volume = $a^3 = (8)^3 = 512 \text{ cm}^3$

Question 4.
A cube of edge 6 cm and a cuboid with dimensions 4 cm x x cm x 15 cm are equal in volume. Find:
(i) the value of x.
(ii) total surface area of the cuboid.
(iii) total surface area of the cube.
(iv) which of these two has greater surface and by how much?
Solution:
Edge of a cube = 6 cm
Volume = $a^3 = (6)^3 = 216 \text{ cm}^3$
Dimensions of a cuboid = 4 cm x x cm x 15 cm
Volume = $60x \text{ cm}^3$
Volume of both is equal

(i) $\therefore 60x = 216 \Rightarrow x = \frac{216}{60} = \frac{36}{10}$

$\therefore x = 3.6$ cm

(ii) Total surface area of cuboid
$= 2[lb + bh + hl]$
$= 2[4 \times 3.6 + 3.6 \times 15 + 15 \times 4] \text{ cm}^2$
$= 2[14.4 + 54.0 + 60] \text{ cm}^2$
$= 128.4 \times 2 = 256.8 \text{ cm}^2$

(iii) Total surface area of cube
$= 6a^2 = 6(6)^2 = 6 \times 36 = 216 \text{ cm}^2$

(iv) Difference of surface areas $= 256.8 - 216$
$= 40.8 \text{ cm}^2$
$\therefore$ Surface area of cuboid is greater

Question 5.
The capacity of a rectangular tank is 5.2 m^3 and the area of its base is 2.6 x 104 cm^2; find its height (depth).
Solution:
Capacity of a tank = 5.2 m^3
and area of its base = 2.6 x 10^4 cm^2

$$= \frac{2.6 \times 10000}{100 \times 100} = 2.6 \text{ m}^2$$

$$\Rightarrow lb = 2.6 \text{ m}^2$$

$$\text{and } lbh = 5.2 \text{ m}^3$$

$$\therefore \text{ Height } (h) = \frac{5.2}{2.6} = 2 \text{ m}$$

Question 6.
The height of a rectangular solid is 5 times its width and its length is 8 times its height. If the volume of the wall is 102.4 cm^3, find its length.
Solution:
Height of rectangular solid = 5 x width
and length = 8 x height = 8 x 5 x width = 40 x width
Volume = 102.4 cm^3
Let width = w
Then height = 40w
and height = 5w

$$\therefore w \times 40w \times 5w = 102.4$$

$$w^3 = \frac{102.4}{40 \times 5} = 0.512$$

$$= (0.8)^3$$

$$\therefore w = 0.8$$

$$\therefore \text{ Length} = 40w = 40 \times 0.8 = 32 \text{ cm}$$

Question 7.
The ratio between the lengths of the edges of two cubes are in the ratio 3 : 2. Find the ratio between their:
(i) total surface area
(ii) volume.
Solution:
Ratio in edges of two cubes = 3:2
Let edge of first cube = 3x
Then edge of second cube = 2x
(i) Now total surface area of first cube = 6 x $(3x)^2$ = 6 x $9x^2$ = $54x^2$
and of surface area of second cube = 6 x $(2x)^2$ = 6 x $4x^2$ = $24x^2$

Ratio = $54x^2 : 24x^2$ = 9:4
(ii) Volume of first cube = $(3x)^3 = 27x^3$
and second cube = $(2x)^3 = 8x^3$
Ratio = $27x^3 : 8x^3$ = 27 :8

Question 8.
The length, breadth and height of a cuboid (rectangular solid) are 4 : 3 : 2.
(i) If its surface are is 2548 cm^2, find its volume.
(ii) If its volume is 3000 m^3, find its surface area.
Solution:
Surface area of cuboid = 2548 cm^2
Ratio in length, breadth and height of a cuboid = 4 : 3 : 2
Let length = 4x, Breadth = 3x and height = 2x

$\therefore$ Surface area $= 2(4x \times 3x + 3x \times 2x + 2x \times 4x)$
$= 2(12x^2 + 6x^2 + 8x^2)$
$= 2 \times 26x^2 = 52x^2$
$\therefore 52x^2 = 2548$

$$x^2 = \frac{2548}{52} = 49 = (7)^2$$

$\therefore x = 7$
$\therefore$ Length $= 4x = 4 \times 7 = 28$ cm
Breadth $= 3x = 3 \times 7 = 21$ cm
and height $= 2x = 2 \times 7 = 14$ cm
$\therefore$ Volume $= lbh$
$= 28 \times 21 \times 14 \text{ cm}^3 = 8232 \text{ cm}^2$
(*ii*) If volume = 3000 m^3
$\Rightarrow 4x \times 3x \times 2x = 3000$
$\Rightarrow 24x^3 = 3000$

$$\Rightarrow x^3 = \frac{3000}{24} = 125 = (5)^3$$

$\therefore x = 5$ m
Length $= 5 \times 4 = 20$, breadth $= 5 \times 3 = 15$ m
and height $= 5 \times 2 = 10$ m
$\therefore$ Surface area $= 2[lb + bh + hl]$
$= 2[20 \times 15 + 15 \times 10 + 10 \times 20] \text{ m}^2$
$= 2\,[300 + 150 + 200] \text{ m}^2$
$= 2 \times 650 = 1300 \text{ m}^2$

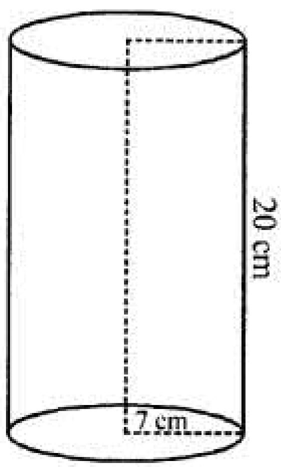
EXERCISE 21(D)

Question 1.

The height of a circular cylinder is 20 cm and the diameter of its base is 14 cm. Find:
(i) the volume
(ii) the total surface area.

Solution:

Height of cylinder (h) = 20 cm
and diameter of its base (d)= 14 cm
and radius of its base (r)= 14/2 = 7 cm
(i) Volume = $\pi r^2 h$
= 22/7 x 7 x 7 x 20 cm^3 = 3080 cm^3



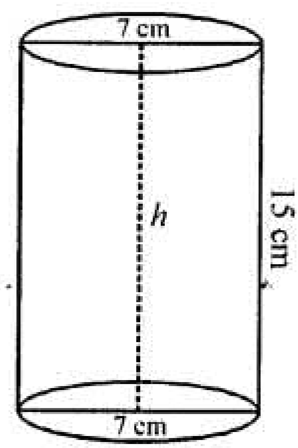
(ii) Total surface area = $2\pi r(h + r)$
= 2 x 22/7 x 7 (20 + 7) cm^2 = 44 x 27 = 1188 cm^2

Question 2.

Find the curved surface area and the total surface area of a right circular cylinder whose height is 15 cm and the diameter of the cross-section is 14 cm.

Solution:

Diameter of the base of cylinder = 14 cm
Radius (r) = 14/2 cm = 7 cm
Height (h) = 15 cm



Curved surface area = 2πrh
= 2 x 22/7 x 7 x 15 = 660 cm^2
Total surface area = 2πr (h + r)
= 2 x 22/7 x 7(15 + 7)
= 2 x 22/7 x 7 x 22 = 968 cm^2

Question 3.
Find the height of the cylinder whose radius is 7 cm and the total surface area is 1100 cm^2.
Solution:
Total surface area =1100 cm^2
Radius = 7 cm
Let height of the cylinder = h
Then, total surface area = 2πr(h + r)

$$\Rightarrow 2 \times \frac{22}{7} \times 7(h + 7) = 1100$$

$$\Rightarrow 44(h + 7) = 1100$$

$$\Rightarrow 44h + 308 = 1100$$

$$\Rightarrow 44h = 1100 - 308$$

$$h = \frac{792}{44} = 18 \text{ cm}$$

Question 4.
The curved surface area of a cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.
Solution:
Height (h) = 14 cm
Curved surface area (2πrh) = 88 cm^2

Then, $2\pi rh = 88\ cm^2$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88\ cm^2$$

$$\Rightarrow 88r = 88$$

$$\Rightarrow r = \frac{88}{88} = 1\ cm$$

Then diameter = $1 \times 2 = 2$ cm

Question 5.
The ratio between the curved surface area and the total surface area of a cylinder is 1 : 2. Find the ratio between the height and the radius of the cylinder.
Solution:
Let r be the radius and h be the height of a right circular cylinder, then Curved surface area = $2\pi rh$
and total surface area = $2\pi rh \times 2\pi r^2 = 2\pi r(h + r)$
But their ratio is 1 : 2

$$\therefore \frac{2\pi r}{2\pi r(h+r)} = \frac{1}{2} \Rightarrow \frac{h}{h+r} = \frac{1}{2}$$

$$\Rightarrow 2h = h + r \Rightarrow 2h - h = r$$

$$\Rightarrow h = r = 1 : 1$$

Hence their radius and height are equal.

Question 6.
Find the capacity of a cylindrical container with internal diameter 28 cm and height 20 cm.
Solution:
Diameter = 28 cm
Radius = 28/2 cm = 14 cm
Height = 20 cm
Volume = $\pi r^2 h$ = 22/7 x 14 x 14 x 20
Volume = 12320 cm^3

Question 7.
The total surface area of a cylinder is 6512 cm^2 and the circumference of its bases is 88 cm. Find:
(i) its radius
(ii) its volume
Solution:
Let r be the radius and h be the height of the given cylinder.
Circumference = $2\pi r$ = 88 cm (Given)

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88 \text{ cm}$$

$$\Rightarrow r = 88 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow r = 14 \text{ cm}$$

Total surface area $= 2\pi r(h + r) = 6512 \text{ cm}^2$ (Given)

$$\Rightarrow 88(h + 14) = 6512$$

$(\because 2\pi r = 88 \text{ cm and } r = 14 \text{ cm})$

$$\Rightarrow h + 14 = \frac{6512}{88}$$

$$\Rightarrow h + 14 = 74 \Rightarrow h = 60 \text{ cm}$$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (14)^2 \times 60 \text{ cm}^3 = 36960 \text{ cm}^3$$

Question 8.
The sum of the radius and the height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm^2. Find the height and the volume of the cylinder.
Solution:
Let r and h be the radius and height of the solid cylinder respectively.
Given, r + h = 37 cm
Total surface area of the cylinder = 1628 cm^2 (Given)

$$\therefore 2\pi r(r + h) = 1628 \text{ cm}^2$$

$$\Rightarrow 2\pi r \times 37 = 1628 \text{ cm}^2$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 37 = 1628 \text{ cm}^2$$

$$\Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7 \text{ cm}$$

$r + h = 37 \text{ cm} \Rightarrow 7 + h = 37 \text{ cm} \Rightarrow h = 30 \text{ cm}$

Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

Question 9.
A cylindrical pillar has radius 21 cm and height 4 m. Find :
(i) the curved surface area of the pillar
(ii) cost of polishing 36 such cylindrical pillars at the rate of Rs. 12 per m^2.
Solution:
Radius of the cylinder = 21 cm

$$= \frac{21}{100} = 2.1 \text{ m}$$

Height of the cylinder = 4 m

(*i*) Curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4 = 52.8 \text{ m}^2$$

(*ii*) Cost of polishing 1 m^2 = ₹12
Cost of polishing (36×52.8 m^2)
= ₹12 × 1900.80 = ₹22,809.60

Question 10.
If radii of two cylinders are in the ratio 4 : 3 and their heights are in the ratio 5 : 6, find the ratio of their curved surfaces.
Solution:
Ratio in radii of two cylinders = 4 : 3
and ratio in their heights = 5 : 6

Let r_1 and r_2 be the radii and h_1, h_2 be their heights respectively.

$\therefore r_1 : r_2 = 4 : 3$ and $h_1 : h_2 = 5 : 6$

$\therefore r_1 = \frac{4}{3}$ and $\frac{h_1}{h_2} = \frac{5}{6}$

$\therefore$ Surface area of the first cylinder = $2\pi r_1 h_1$
and area of second cylinder = $2\pi r_2 h_2$

$$\frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1}{r_2} \times \frac{h_1}{h_2} = \frac{4}{3} \times \frac{5}{6} = \frac{20}{18}$$

$$= \frac{10}{9} = 10 : 9$$

$\therefore$ Ratio in their surface areas = 10 : 9

EXERCISE 21(E)

Question 1.
A cuboid is 8 m long, 12 m broad and 3.5 high, Find its
(i) total surface area
(ii) lateral surface area
Solution:
Length of a cuboid = 8 m
Breadth of a cuboid = 12 m
Height of a cuboid = 3.5 m
(i) Total surface area = 2(lb + bh + hl)
= 2(8 x 12 + 12 x 3.5 + 3.5 x 8)
= 2(96 + 42 + 28)
= 2 x 166 = 332 m^2
(ii) Lateral surface area = 2h(l + b)
= 2 x 3.5(8 + 12) = 7 x 20= 140 m^2

Question 2.
How many bricks will be required for constructing a wall which is 16 m long, 3 m high and 22.5 cm thick, if each brick measures 25 cm x 11.25 cm x 6 cm ?
Solution:
Length of the wall = 16 m = 16 x 100 cm = 1600 cm
Height of the wall = 3 m = 3 x 100 cm = 300 cm
Breadth of the wall = 22.5 cm
Volume of the wall = 1600 x 300 x 22.5 cm3 = 1,08,00,000 cm^3
Volume of one brick = 25 x 11.25 x 6 cm3 = 1687.5 cm^3
Number of bricks required to construct the

$$\text{wall} = \frac{\text{Volume of wall}}{\text{Volume of one brick}}$$

$$= \frac{1{,}08{,}00{,}000}{1687{\cdot}5} = 6400$$

Question 3.
The length, breadth and height of cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2, find its volume.
Solution:
Let length of the given cuboid = 6x
Breadth of the given cuboid = 5x
Height of the given cuboid = 3x

Total surface area of the given cuboid

$= 2(lb + bh + hl)$

$= 2(6x \times 5x + 5x \times 3x + 3x \times 6x)$

$= (30x^2 + 15x^2 + 18x^2)$

$= 2 \times 63x^2 = 126x^2$

But, we are given total surface area

$= 504\ cm^2$

$\therefore\ 126x^2 = 504$

$\Rightarrow x^2 = \frac{504}{126}$

$\Rightarrow x^2 = 4$

$\Rightarrow x^2 = (2)^2$

$\Rightarrow x = 2\ cm$

$\therefore$ Length of the given cuboid $= 6x$

$= 6 \times 2\ cm = 12\ cm$

Breadth of the given cuboid $= 5x$

$= 5 \times 2\ cm = 10\ cm$

Height of the given cuboid $= 3x$

$= 3 \times 2\ cm = 6\ cm$

Now, volume of the cuboid $= l \times b \times h$

$= 12 \times 10 \times 6 = 720\ cm^3$

Question 4.

The external dimensions of an open wooden box are 65 cm, 34 cm and 25 cm. If the box is made up of wood 2 cm thick, find the capacity of the box and the volume of wood used to make it.

Solution:

External length of the open box = 65 cm
External breadth of the open box = 34 cm
External height of the open box = 25 cm
External volume of the open box = 65 x 34 x 25 cm^3 = 55250 cm^3
Internal length of open box = 65 – (2 x 2) cm = 61 cm
Internal breadth of a open box = 34 – (2 x 2) cm = 30 cm
Internal height of open box = 25 – 2 cm = 23 cm
Internal volume of open box or capacity of the box = 61 x 30 x 23 cm^3 = 42090 cm^3
Volume of wood required to make the closed box = 55250 – 42090 cm^3 = 13160 cm^3

Question 5.

The curved surface area and the volume of a toy, cylindrical in shape, are 132 cm^2 and 462 cm^3respectively. Find, its diameter and its length.

Solution:
Let the radius of a toy = r and
height of the toy = h
Curved surface area of a toy = 132 cm^2
=> $2\pi rh = 132\ cm^2$

$$\Rightarrow 2\pi rh = 132\ cm^2$$

$$\Rightarrow r = \frac{132}{2\pi \times h} cm^2 \quad ...(i)$$

Also, volume of a toy = 462 cm^3

$$\Rightarrow \pi r^2 h = 462\ cm^3$$

$$\Rightarrow r^2 = \frac{462}{\pi \times h} \quad ...(ii)$$

Now, substitute the volume of r, we get

$$\frac{(132)^2}{(2)^2 \times (\pi)^2 \times h^2} = \frac{462}{\pi \times h}$$

$$\Rightarrow \frac{132^2}{4 \times \pi \times h} = 462$$

$$\Rightarrow 4 \times \pi \times h = \frac{132 \times 132}{462}$$

$$\Rightarrow h = \frac{132 \times 132}{462 \times \pi \times h}$$

$$\Rightarrow h = \frac{132 \times 132 \times 7}{462 \times 22 \times 4} = 3\ cm$$

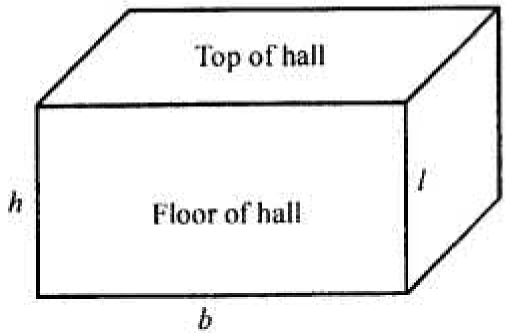
Now, put the value of h in eq. (i), we get

$$r = \frac{132 \times 7}{2 \times 22 \times 3} = 7\ cm$$

$\therefore$ Diameter of the toy = $2 \times r$
$= 2 \times 7\ cm = 14\ cm$

Question 6.
The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15,000, find the height of the hall.
Solution:

Let length, breadth and height of the rectangular hall be l m, b m and h m respectively.



Perimeter of the floor of hall = $2(l + b)$

$250 \text{ m} = 2(l + b)$

$$(l + b) = \frac{250}{2} = 125 \text{ cm} \qquad \ldots(i)$$

Area of four walls = Area of cuboid – Area of floor – Area of top
= 2 (lb + bh + hl) – (l x b) – (l x b)
= 2(lb) + 2 (bh) + 2(hl) – 2lb = 2 lh + 2 bh
= 2h(l + b)
= 2h x 125 [From (i)]
= 250h m^2
Area of four walls = 250h m^2
Cost of painting 1 m^2 area = Rs. 10
Cost of painting 250h m^2 area = Rs. 10 x 250h = 2500h
15000 = 2500h
h = 15000/2500
The height of the hall is 6 m.

Question 7.
The length of a hall is double its breadth. Its height is 3 m. The area of its four walls (including doors and windows) is 108 m^2, find its volume.
Solution:
Let the breadth be x
and the length be 2x
Height = 3 m

Area of four walls = 108 m^2

$\Rightarrow 2(l + b)h = 108$

$\Rightarrow 2(2x + x)3 = 108$

$\Rightarrow 6 \times 3x = 108$

$\Rightarrow 3x = \frac{108}{6}$

$\Rightarrow x = \frac{18}{3} = 6$ m

$\therefore$ Breadth = x = 6 m

and length = $2x$ = 12 m

Hence, Volume = $l \times b \times h$

$= 12 \times 6 \times 3$

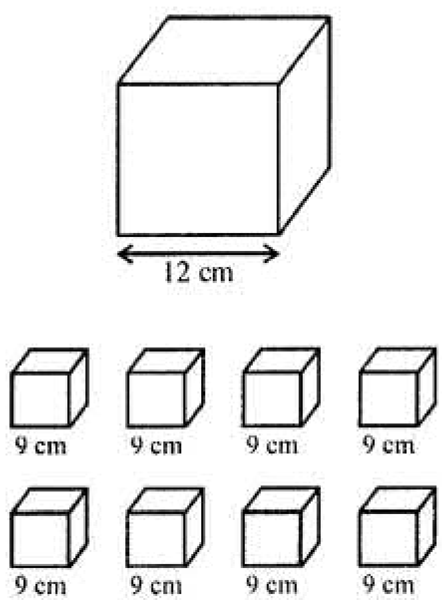
$= 216$ m^3

Question 8.

A solid cube of side 12 cm is cut into 8 identical cubes. What will be the side of the new cube? Also, find the ratio between the surface area of the original cube and the total surface area of all the small cubes formed.

Solution:

Here, cube of side 12 cm is divided into 8 cubes of side 9 cm.



Given that,

Their volumes are equal.

Volume of big cube of 12 cm = Volume of 8 cubes of side a cm

$(\text{Side of big cube})^3 = 8 \times (\text{Side of small cube})^3$

$(12)^3 = 8 \times a^3$

$$\Rightarrow a^3 = \frac{12 \times 12 \times 12}{8}$$

$$\Rightarrow a^3 = 6^3 \text{ cm}^3$$

$$\Rightarrow a = 6 \text{ cm}$$

$\therefore$ Side of small cube = 6 cm

Ratio of their surface

$$= \frac{\text{Surface area of the original cube}}{\text{Total surface area of the small cube}}$$

$$= \frac{6(\text{side of big cube})^2}{8 \times 6(\text{side of small cube})^2}$$

$$= \frac{6 \times 12 \times 12}{8 \times 6 \times 6 \times 6} = \frac{4}{8} = 1 : 2$$

So, the ratio is 1 : 2

Question 9.
The diameter of a garden roller is 1.4 m and it 2 m long. Find the maximum area covered by it 50 revolutions?

Solution:
Diameter of the roller = 1.4 m
Radius (r) = 1.4/2 = 0.7 m
and length (h) = 2m
Curved surface area = 2πrh = 2 x 22/7 x 0.7 x 2 cm^2 = 8.8 m^2
Area covered in 50 complete revolutions = 8.8 x 50 m^2 = 440 m^2
Area of the playground = 440 m^2

Question 10.
In a building, there are 24 cylindrical pillars. For each pillar, radius is 28 m and height is 4 m. Find the total cost of painting the curved surface area of the pillars at the rate of ₹ 8 per m^2.

Solution:
Radius (r) of each pillar = 28 m
Height (h) = 4 m
Curved surface area of each pillar = 2πrh
= 2 x 22/7 x 28 x 4 m^2 = 704 m^2
Surface area of 24 pillars = 704 x 24 m^2 = 16,896 m^2
Cost of cleaning = Rs. 8 per m^2
Total cost = Rs. 16,896 x 8 = Rs. 1, 35, 168